KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

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Solution Sheet 4

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Solution 1

1. Explain the terms Voronoi region, Voronoi edge and Voronoi vertex:

   Voronoi region: A Voronoi region (also called Voronoi cell) includes all points of the Euclidean space that are closer to a given point in P than to any other point in P.

   Voronoi edge: The Voronoi edges mark the boundaries of the Voronoi regions. All points of a Voronoi edge have the same distance to the centers of the adjacent regions, with the center of a region corresponding to a point in P.

   Voronoi vertex: A Voronoi vertix is a corner of a Voronoi region (intersection of several Voronoi edges).

2. Determine the Voronoi diagram for P:

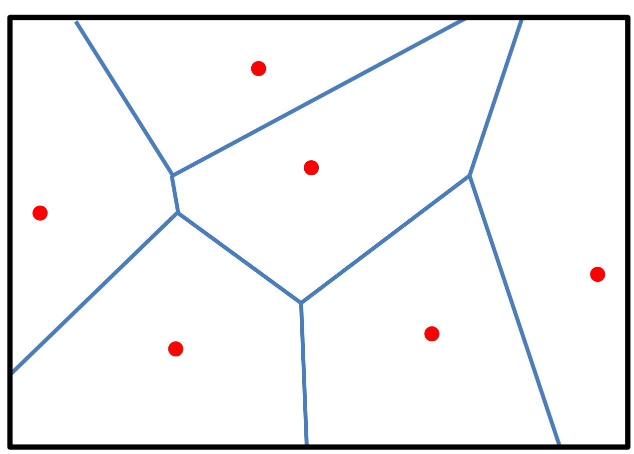
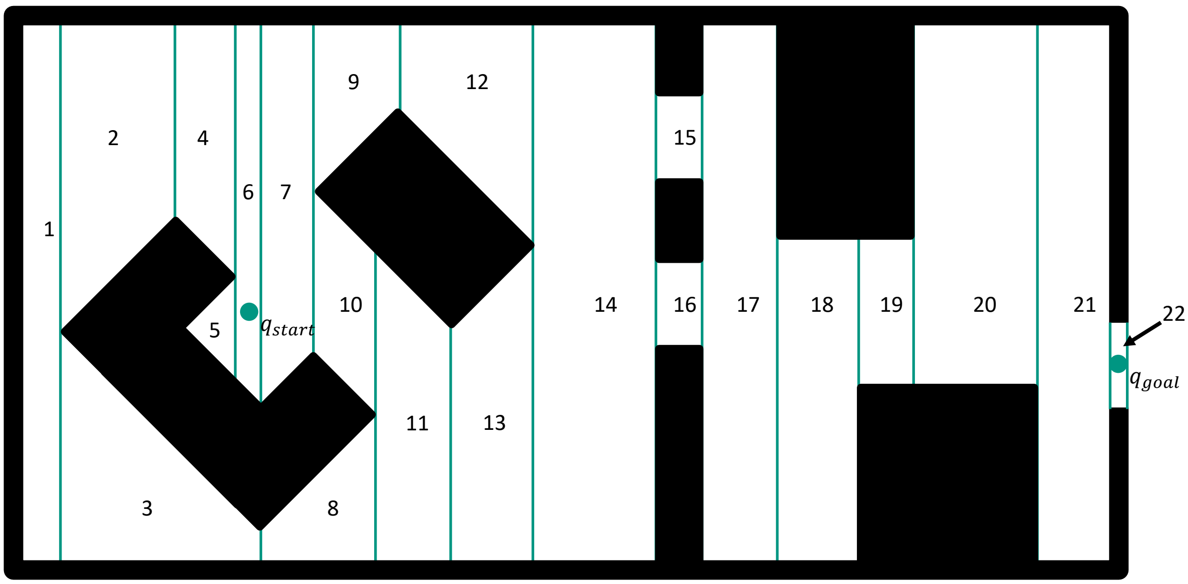


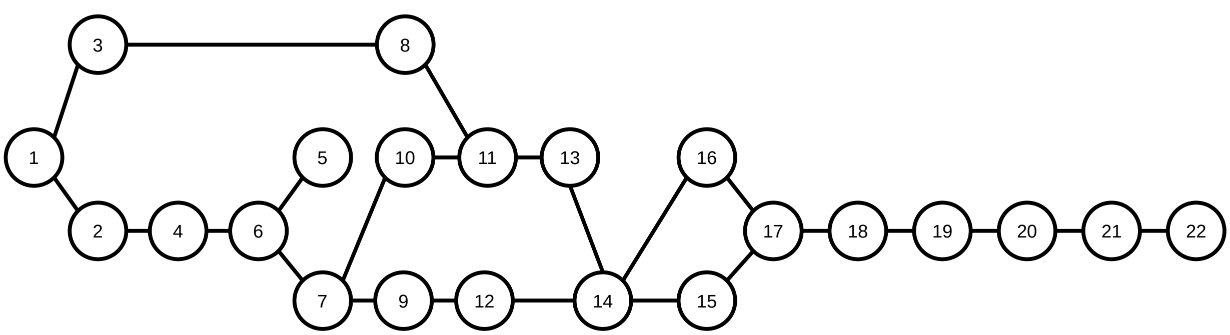
Figure 1: Voronoi diagram for the point set P.

Solution 2

1. Cell decomposition using Line-Sweep:



2. Adjacency graph:



3. Possible sequence of cells that are passed through from q_{start} to q_{goal}:
   6, 7, 9, 12, 14, 15, 17, 18, 19, 20, 21, 22

Solution 3

1. The node q_{new} was determined using the RRT* function $Steer(q_{nn},q_s,d)$. q_s is the current sample, d is the step size and q_{nn} is the nearest neighbor of q_s, in this case q_7. q_{new} is obtained by performing a step of length d from q_7 to q_s.

2.

Node	Path costs
q_1	0
q_2	1
q_3	3
q_4	4
q_5	6
q_6	7
q_7	7
q_8	10
q_9	10
q_{new}	8

3. The RRT* function $Near(T,q_{new},r)$ determines all nodes from T whose distance to q_{new} is at most r.

4. For the *Rewire* step of the RRT* algorithm, the nodes q_5, q_7, q_8 and q_9 are taken into account, as they were determined using the *Near* function (q_5 and q_7 can be ignored as these nodes were already considered during the *MinCostPath* step).

5. The outcome of the *Rewire* step can be seen in Figure 2. The connection to q_8 is adjusted because $Cost(q_{new}) + Cost(q_{new}, q_8) = 9$ is lower than $Cost(q_8) = 10$. The connection q_9 is adjusted because $Cost(q_{new}) + Cost(q_{new}, q_9) = 9$ is lower than $Cost(q_9) = 10$. The connections to q_7 and q_5 remain as the costs do not decrease.

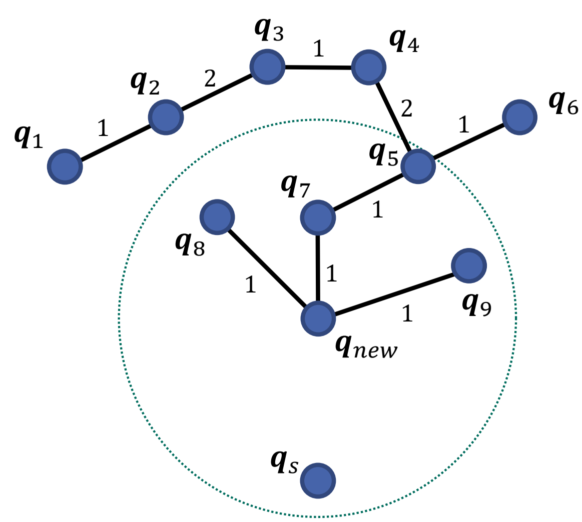


Figure 2: The RRT* tree T after the *Rewire* step.

Solution 4

1. In the following, the first four steps of the A* algorithm are shown. The fourth step was not required, but indicates the state of the Open Set and the Closed Set after the third step.

Step 1

- Open Set: $\{v_2(cost : 4.12, pred : (null))\}$
- Closed Set: $\{\}$
- Node to be expanded: v_2
- Nodes that are added to the Open Set:
  $v_3(cost : 1.00 + 4.47, pred : v_2)$
  $v_1(cost : 1.00 + 4.00, pred : v_2)$
  $v_5(cost : 4.00 + 3.16, pred : v_2)$

Step 2

- Open Set: $\{v_3(cost : 5.47, pred : v_2), v_1(cost : 5.00, pred : v_2), v_5(cost : 7.16, pred : v_2)\}$
- Closed Set: $\{v_2\}$
- Node to be expanded: v_1
- Nodes that are added to the Open Set:
  $v_4(cost : 2.00 + 3.00, pred : v_1)$

Step 3

- Open Set: $\{v_3(cost : 5.47, pred : v_2), v_5(cost : 7.16, pred : v_2), v_4(cost : 5.00, pred : v_1)\}$
- Closed Set: $\{v_1, v_2\}$
- Node to be expanded: v_4
- Nodes that are added to the Open Set:
  $v_7(cost : 6.00 + 2.00, pred : v_4)$

Step 4

- Open Set: $\{v_3(cost : 5.47, pred : v_2), v_5(cost : 7.16, pred : v_2), v_7(cost : 8.00, pred : v_4)\}$
- Closed Set: $\{v_1, v_2, v_4\}$
- Node to be expanded: v_3
- Nodes that are added to the Open Set:
  $v_6(cost : 2.00 + 3.61, pred : v_3)$

2. When selecting the heuristic for the A* algorithm, it is important to ensure that it does not overestimate the remaining costs to the goal. Otherwise, the guarantee that the shortest path to the goal will be found no longer applies. In this exercise, the Euclidean distance is always less than or equal to the actual minimum cost to the goal, as there are no nodes with costs < 1.

3. When the target node is added to the Open Set, the A* algorithm cannot terminate as it is still possible that shorter paths to the target node exist. If the target node is to be expanded, the shortest path is found so that the algorithm can terminate.

Solution 5

1. Potential fields act on a robot if the distance between the center of the field and the robot is less than ρ_0.

   - $\|\mathbf{q}_R - \mathbf{q}_{\text{rep},1}\| \approx 2.2 < \rho_0$
   - $\|\mathbf{q}_R - \mathbf{q}_{\text{rep},2}\| \approx 1.4 < \rho_0$
   - $\|\mathbf{q}_R - \mathbf{q}_{\text{rep},3}\| = 1 < \rho_0$

   At the given robot position, all three repulsive potentials act on R.

2. In the given case, four potentials act on the robot: three repulsive potentials and the attracting potential. The FIRAS function for a repulsive potential $U_{\text{rep},*}(\mathbf{q}_R)$ is defined as follows ($\nu = 1$):

   $$U_{\text{rep},*}(\mathbf{q}_R) = \frac{1}{2}\left(\frac{1}{\|\mathbf{q}_R - \mathbf{q}_{\text{rep},*}\|} - \frac{1}{\rho_0}\right)^2$$

   The linear function for the attracting target potential is ($k = 1$):

   $$U_{\text{attr}}(\mathbf{q}_R) = \|\mathbf{q}_R - \mathbf{q}_{\text{goal}}\|.$$

   The following expression results for the function $U(\mathbf{q}_R)$ of the acting potentials:

   $$\begin{aligned} U(\mathbf{q}_R) &= U_{\text{attr}}(\mathbf{q}_R) + \sum_{i=1}^{3} U_{\text{rep},i}(\mathbf{q}_R). \\ &= \|\mathbf{q}_R - \mathbf{q}_{\text{goal}}\| + \sum_{i=1}^{3} \frac{1}{2}\left(\frac{1}{\|\mathbf{q} - \mathbf{q}_{\text{rep},i}\|} - \frac{1}{\rho_0}\right)^2 \end{aligned}$$

3. The robot moves in the direction of the force $F(\mathbf{q}_R)$:

   $$\begin{aligned} F(\mathbf{q}_R) &= -\nabla U(\mathbf{q}_R) \\ &= -\nabla\left(U_{\text{attr}}(\mathbf{q}_R) + \sum_{i=1}^{3} U_{\text{rep},i}(\mathbf{q}_R)\right) \\ &= -\nabla U_{\text{attr}}(\mathbf{q}_R) + \sum_{i=1}^{3} -\nabla U_{\text{rep},i}(\mathbf{q}_R) \\ &= -\frac{\mathbf{q}_R - \mathbf{q}_{\text{goal}}}{\|\mathbf{q}_R - \mathbf{q}_{\text{goal}}\|} + \sum_{i=1}^{3}\left(\frac{1}{\|\mathbf{q}_R - \mathbf{q}_{\text{rep},i}\|} - \frac{1}{\rho_0}\right) \cdot \frac{1}{\|\mathbf{q}_R - \mathbf{q}_{\text{rep},i}\|^2} \cdot \frac{\mathbf{q}_R - \mathbf{q}_{\text{rep},i}}{\|\mathbf{q}_R - \mathbf{q}_{\text{rep},i}\|} \end{aligned}$$

   With the values given in the exercise, the force acting on the robot can be calculated as follows:

$$
\begin{aligned}
F(\mathbf{q}_R) = \quad & \begin{pmatrix} 1 \\ 0 \end{pmatrix} & & + \left(\frac{1}{2.2} - \frac{1}{5}\right) \cdot \frac{1}{2.2^3} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\
& & & + \left(\frac{1}{1.4} - \frac{1}{5}\right) \cdot \frac{1}{1.4^3} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} \\
& & & + \left(1 - \frac{1}{5}\right) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
= \quad & \begin{pmatrix} 1 \\ 0 \end{pmatrix} & & + \begin{pmatrix} 0.02 \\ 0.05 \end{pmatrix} + \begin{pmatrix} -0.18 \\ 0.18 \end{pmatrix} + \begin{pmatrix} 0.8 \\ 0 \end{pmatrix} \\
= \quad & \begin{pmatrix} 1.64 \\ 0.23 \end{pmatrix}
\end{aligned}
$$

The interaction of the potentials causes the robot R to move along a curve with the current direction $\mathbf{d} = \begin{pmatrix} 1.64 \\ 0.23 \end{pmatrix}$ instead of the direct path to the goal $\mathbf{q}_{\mathrm{goal}}$.